# Chapter 6

# Culture and Mathematics in School: Boundaries Between "Cultural" and "Domain" Knowledge in the Mathematics Classroom and Beyond

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This chapter is about culture and mathematics teaching and learning. Our goal is to offer a thoughtful treatment of the role of culture in the teaching and learning of mathematics and to synthesize literature that is relevant to this concern from multiple subdisciplines in education, including math education, educational anthropology, sociology, sociolinguistics, and critical theory. As we do so, we will consider boundaries between what is commonly thought of as "cultural" knowledge (that is, knowledge derived from settings outside of school, typically in students' homes and communities) and "domain" knowledge (that is, knowledge valued in the practices prescribed by mathematicians and math educators). Of course, in reality, all knowledge is cultural. All knowledge is related to our experience in the social and cultural worlds that we inhabit, and all knowledge comes to us as it passes through social and cultural systems and institutions through the socializing of norms, values, conventions, and practices. Some have even argued that the dichotomy between "everyday" and "school" mathematics is false (Moschkovich, 2007).

It is also true that knowledge is not neutral with respect to power—some types of knowledge are more aligned with communities of practice that hold more power, whereas other types of knowledge are more aligned with communities of practice that have less power. When viewed through this lens, any discussion of boundaries between mathematical knowledge and cultural knowledge must respect that these issues of power are implicated in our definitions, issues of concern, and the very

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conversation in which we are engaged through our scholarship. Furthermore, knowledge is fundamentally tied to the kinds of people we (and others) view ourselves to be and the trajectory we (and others) view ourselves to be on. In other words, issues of identity are critical to understanding both the development of mathematical knowledge for individuals and communities but also to considering how we draw lines between cultural and domain knowledge.

We would like to begin with a story about the experience of cultural and domain mathematics for one group of African American students in California. For us, this story illustrates important themes in understanding the cultural nature of mathematics learning and tensions to consider in elucidating boundaries between domain and cultural knowledge in mathematics. As a part of a study on thinking and learning across contexts (Nasir, 1996, 2000) middle and high school basketball players were asked to solve average and percentage problems two ways: In one set of tasks, the problems were framed by the practices of basketball, and in the other, problems were given in the format of a typical school math worksheet. Two examples of problems in both formats are presented in Table 1.

Players' responses to these two sets of problems were striking. Overall, players were better able to solve the problems in the context of basketball, and they used quite different strategies across the two contexts. On basketball problems, players tended to use invented strategies, such as a strategy for computing the average that involved adding and subtracting between the numbers until they were all the same or a strategy for calculating percentages that involved assuming each shot was worth 10% (so that the problem above would be 70% because the players made seven shots). On school math problems, players used algorithms, often misremembered, to manipulate numbers. How do we think about which of these kinds of responses are more mathematically sound? Through one lens, the use of algorithms is a powerful and concise way to solve math problems involving average and percentage and speaks to one's ability to leverage the collective wisdom in the field for a reliable and tidy solution. However, players often misapplied the algorithms and may not have understood the mathematics behind the operations that they performed. Through another lens, the invented strategies (in some cases) show understanding of fundamental mathematical principles. The average strategy presented above is founded on an intuitive understanding of the principle of an average (a number that represents a group of numbers). However, other strategies, although useful for some problems, could lead to mathematically incoherent solutions on other problems. For instance, the percentage strategy cited above is a mathematically inflexible solution path for generalization across different problem contexts. Furthermore, the point can also be made that the very problems students are solving across these contexts are different in nature. On the basketball problems, students were reasoning about discrete quantities—quantities that had shape and form in the real world. The school problems asked students not to reason about quantities directly but to work with symbolic representations of quantities. We argue, though, the differences in students' solutions were not solely because of the presence or absence of symbolic representations;

**TABLE 1 School and Basketball Format Problems**

| Problem Type | Basketball Format | School Format |
|---|---|---|
| Calculating a percentage | "Say you are at the free-throw line. You take 11 shots and you make seven of them. What's your percentage from the line?" | 7/11 = ____ % |
| Calculating the average of a series of numbers | "In the first game of the season, you score 15 points. In the second game you score 20 points. In the third game you score 10 points. What is your average score for those three games?" | Students were shown a list of the numbers with a blank box in which to write the average of (15, 20, 10). Instructions were written as "Calculate the average for these sets of numbers and write the solution in the box." |

rather, they reflect differences in students' sense of themselves and their abilities in these settings.

Basketball players' patterns of solutions and strategies illustrate what is often a discontinuity between students' everyday cultural knowledge about math and the type of mathematics instruction and classroom activities many students are exposed to in school. The observed response patterns also point to one way that culture can become salient (even if it is not recognized as so) in the math classroom. That is, the basketball players possessed knowledge about average and percentage that was inaccessible in the math classroom and that their teachers likely did not know that they had.

Thus far, our analysis of players' solution patterns across problems has been primarily cognitive. However, the findings also speak to the sociocultural aspects of the boundaries between domain and cultural knowledge in math. When players were asked to solve the basketball problems first, they did better on all of the problems. When they were asked to solve the school problems first, they scored lower on all of the problems. This order effect has implications for considering what these two problem sets may have indexed beyond mathematical knowledge. It may be that players who solved the school problems first experienced relative failure and incompetence, which rendered them unable to call up complex reasoning strategies when they solved the basketball problems. When players got the basketball problems first, they took up positions as knowledgeable experts and were emotionally equipped to take on both sets of problems. Interestingly, getting the school problems first left the boundaries between cultural and domain knowledge intact, whereas getting the basketball problems first may have begun to blur these boundaries. Students' sense of themselves as mathematical thinkers and capable learners are at play here. This sense of themselves is rooted in students' histories of participation in mathematics in basketball and in school mathematics.

This analysis raises questions about what counts as mathematical knowledge and productive mathematical activity. It also points to the importance of discerning how the features of different social contexts, in interaction with the proclivities and dispositions of students, mediate what is learned. This dual lens is critical in that learning occurs at the intersection of individual learners (their preferences, sensibilities, and histories of participation in math classrooms) and social contexts with sets of norms and conventions for engagement, availability of supports, and assumptions about learners.

In this chapter, we consider relations between cultural and domain knowledge in mathematics, exploring multiple ways that culture has been viewed by scholars as having relevance to math teaching and learning. Underlying our discussion in this chapter is a concern for the experiences of teaching and learning for nondominant and other marginalized students in American schools, particularly, students who belong to ethnic and social groups currently "underperforming" in mathematics. We draw from Perry, Steele, and Hilliard (2003) in our concern for three kinds of "gaps" in mathematics education: the racial "achievement gap," the gap between potential and achievement for students of color, and the "service gap" widely documented in studies of schools and classrooms across communities. We are all familiar with the racial achievement gap in mathematics and more generally (Haycock, 2001; Secada, 1992; Tate, 1997). White and Asian students continue to outperform African American and Latino students on national tests of mathematics, even when social class is controlled for.[1] This long-standing gap in math achievement is a major national concern and points to the continuing inequities in access to opportunities to learn rich mathematics on multiple levels (Oakes, Joseph, & Muir, 2003). Perry et al. (2003) argue that although this achievement gap by race is important, there are other important "gaps" in education that receive much less attention. First, they argue, there is a gap between "current levels of performance of African [American] students and levels of excellence" (p. 138). In other words, they argue that we know that excellence (not simply adequacy) is in full reach of the masses of African American (and by extension other minority) students, yet many students are not supported to reach this potential for excellence.

Second, they argue that there is quality-of-service gap. They write,

> Nothing is more peculiar than the continuing seeming inability of our leading educators to acknowledge these well-documented savage inequalities and to use them as a basis for explaining the academic, social, and cultural achievements of students. (p. 140)

In pointing out these other two gaps, these authors challenge the mainstream conversation about students of color and about culture and learning, but they also make the argument that reducing all of these gaps is about good teaching.

In this chapter, we take all of these gaps seriously in our discussion of cultural and domain knowledge in mathematics. We consider both theoretical treatments about the relation between culture, race, and math learning, and we also review important contributions with respect to what kinds of practices (both teaching practices and professional development practices) support the reduction of the gaps described above. We also address these issues from a policy perspective, considering the production and reproduction of inequity with respect to math reforms in the past decade.

More specifically, in the second section, we offer our assumptions about knowledge (or knowing) as an inherently cultural activity. In the third section, we briefly explore relations between "everyday" informal math knowledge and school math as a way to enter the conversation about the cultural nature of mathematics. In the fourth section, we attend to the ways in which the field has conceptualized how issues of culture matter in mathematics classes, highlighting three lenses that researchers have used to understand culture and math learning: (a) the way that language mediates knowledge, (b) features of math classrooms as contexts that support or constrain different forms of knowledge, and (c) the way that racialized identities and expectations play out in mathematics classes. In the fifth section, we examine how these issues of culture have taken shape in conversations and research about reforms in mathematics education. In the sixth section, we explore distinct programs and approaches that offer tools and ideas for blurring the line between domain and cultural knowledge in mathematics and briefly reflect on the implications of these issues of culture and math learning for teacher professional development. We conclude by returning to the discussion we began in this introduction—What are the multiple ways that the responses of the basketball players can be interpreted? What might they suggest about cultural and domain knowledge and the empowerment of all students to think and reason mathematically?

## KNOWLEDGE AS CULTURAL ACTIVITY

The view of knowledge we take in this chapter is motivated by our experience both as researchers who study students' acts of cognition and cognizing across a variety of informal and formal contexts for learning and as individuals who are concerned about a system of education that continually tells youth from nondominant groups that they are poor learners. As described above, we have observed some of these youth demonstrating rich mathematical problem-solving strategies in nonschool contexts in a form markedly different from what we typically consider school knowledge. We have also found that these out-of-school environments hold quite different opportunities for youth in terms of authentic problem solving, ongoing feedback, and meaningful relationships (Nasir & Hand, in press). These experiences have led us to reject the notion of knowledge as context independent and thus transportable. Instead, we examine the various forms and functions of knowledge as it is situated in activity. We join with a growing number of educational researchers in the field who conceptualize knowing as both as an in-the-head phenomenon and as constituted in and by cultural practices (Cole & Engeström, 1993; K. D. Gutiérrez, 2002; Kirshner & Whitson, 1997; Moll, 2000; Rogoff, 1990; Rogoff & Lave, 1984).

Central to this view of knowledge is Vygotsky's (1962, 1978b) premise that mental functioning is part and parcel of, even follows, our activity in the social world.

Vygotsky and those who have followed in his stead argue that knowledge is necessarily mediated by tools and signs that we construct and adapt as we coordinate activities with each other to solve problems and achieve our goals (Wertsch, 1991, 1998). We develop goals by assessing what we have the potential to do within a particular context and by negotiating the tools, relationships, and roles that help us carry out our plans in interaction with others (Leontiev, 1978). The past 20 years of research has led us to understand that what one comes to know is necessarily *situated* within socially organized systems of activity (J. S. Brown, Collins, & Duguid, 1989; Cobb & Bowers, 1999; Gibson, 1986; Goodwin, 1981; Greeno & Middle School Math Through Applications Program, 1998; Lave & Wenger, 1991; Rogoff, 1994), *embodied* as individuals project and manage themselves and their goals within these systems (Barsalou, 1999; Varela, Thompson, & Rosch, 1991), and *distributed* through the coordination of informational, material, and interpersonal aspects of these systems over time (Cole & Engeström, 1993; Hutchins, 1995, 1997; Moll, Tapia, & Whitmore, 1993; Pea, 1993). Knowledge in activity, then, or knowing, emphasizes the inextricable links between person and context over interactional history (Cole, 1996; Cole & Scribner, 1974).

Research that takes knowing and coming to know as inseparable focuses on the relation of the individual to the role, position, and patterns of activity that are made available to them as they participate in the practices of various communities (A. L. Brown & Campione, 1994; Lave, 1993; Lave & Wenger, 1991). Researchers have found that students who take their role as learners to be purposeful, integral, and active to the collective enterprise may be more engaged in knowledge-building activities than individuals who simply do what is necessary to succeed (or not to get caught failing) on an immediate task (Engle & Conant, 2002; Nasir, 2002). For example, in the case of the basketball players, participation in the social context of basketball playing required that each individual actively play a role in the execution of a play—their moves being inextricably linked to the moves of others and publicly available for feedback from other players, the coach, and even the fans. Players described how these moves could be broken down into component parts and how they were related both to their overall performance and the success of the team. In the case of a mathematics classroom, however, the players differed in the nature and level of engagement. Some players viewed their role as integral to the mathematics learning of the class by answering the teacher's questions or providing help to their peers. However, other players did not take (nor were they required to take) an active role in the class and instead sat quietly in the background. Lave and Wenger's (1991) model of legitimate peripheral participation explicates how learners may become well practiced at, barely adopt, and even reject the roles and practices of the various communities they encounter. In the mathematics classroom, there was significant opportunity for students to disengage from practices that supported developing mathematical understanding, whereas in the basketball context, high levels of engagement were required by the team.

This perspective challenges the notion, then, that individuals will embrace the opportunities for knowledge development in a learning community in the same way.

Instead, it acknowledges the importance of recognizing the process of *negotiation* that learners undertake as they reconcile new ways of learning and being with the practices and positions they enacted in prior experiences. This process of negotiation forms the contours and texture of their trajectory of participation and necessarily entails issues of power and status. For example, Danny Martin (2006a) poignantly describes the fortitude of adult African American mathematics students who had to reconcile experiences of racism and marginalization around mathematics in grade school with their decision to pursue mathematics in community college many years later. It is also the case that newcomers can challenge their local situations by introducing ways of participating and perceiving into a community of practice that serve to act back on local structures and processes and produce cultural change. We see this occur, for example, in research on professional development and school change where the grassroots initiatives of a small group of teachers can reverberate throughout their district (Dutro, Fisk, Koch, Roop, & Wixson, 2002).

The turn in the field of mathematics education research toward a conceptualization of knowledge as socially situated represents a major shift in thinking about the nature and role of culture in learning. Although there are numerous ways that theorists have conceptualized and operationalized culture (Bishop, 1988; Geertz, 1973; Gonzalez, 1999; Kroeber & Kluckhohn, 1952; Wax, 1993), we draw on a *cultural practice* perspective of culture, with roots in Vygotskian theory of learning, culture, and development (Vygotsky, 1978a). This perspective highlights the culturally organized practices and activities that make up the daily lives of individuals in societies (Lave, 1988; Saxe, 1999; Wertsch, 1998). Such practices are seen as local sites of cultural processes, calling for attention to social interaction, mediation of cognitive processes by tools and artifacts, and multiple interacting levels of context (Cole, 1996; Nasir & Hand, 2006). The metaphor of culture as a "blanket" that surrounds individual cognition in this case is replaced by one as the "fabric" of knowing, where culture and activity are inseparable at the level of individual, group, and societal development. This means that the cultural practices that we engage in as we move across everyday, school, and professional contexts both shape and constitute our learning.

Viewing culture solely in terms of the variations and similarities among practices and orientations misses the role of power in determining which forms of knowledge are considered competent and productive in different contexts. A number of researchers have drawn on Bourdieu's (1977) notion of *cultural capital* to illustrate how a cultural practice, such as school math, is historically and socially reified through broader social structures and processes that privilege certain groups of individuals over time (Cobb & Hodge, 2002; Mehan, Hubbard, & Villanueva, 1994). That some cultural practices are viewed as taken for granted, normative, or independent of their cultural underpinnings is partly a result of the longer time scales of some processes and events (Lemke, 2000; Saxe & Esmonde, 2005) and the tendency of social practices to coalesce into broader, more encompassing constellations. We see this in the way that calculus is presented in today's textbooks, which on the face of it appears static and incontestable. However, the adoption of a Lagrangian perspective of calculus as a set of rigorous algebraic processes versus MacLaurin's perspective of it as geometries and velocities is a decision embedded in a set of conversations that date back to Euler and Newton (Grabiner, 1997). Thus, in addition to locating knowing and doing mathematics within sociocultural activity, and recognizing the differences in how this activity is organized within communities and interpreted by individuals, it is critical to consider how communities (and thus representations and forms of mathematics) come to be privileged over one another.

We perceive the boundaries between cultural and domain knowing and coming to know as being taken up in the research on mathematics education in three ways: (a) mathematics knowing as a cultural activity (the structures and discourse of everyday vs. school math), (b) mathematics learning as a cultural enterprise (the structures and discourse of the classroom vs. students' home and local community), and (c) the system of mathematics education as a cultural system (access to and positioning in the field of mathematics). (See Figure 1.)

This tripartite model of the role of culture in mathematics learning and teaching illustrates how the boundaries between mathematical and cultural knowing are being confronted and examined by researchers at different levels of social activity. These include the cultural entailments of what it means to know mathematics (e.g., basketball vs. classroom mathematical knowing), the cultural entailments of what it means to be and become a mathematics learner within a particular community, and the cultural entailments of what stories get told and decisions made about how, when, and why mathematics is (and is not) learned (e.g., No Child Left Behind [NCLB] vs. the "service gap"). However, though we treat them as analytically distinct for the purposes of this chapter, we recognize that these levels of activity necessarily constitute each other and are reflexively related. In this chapter, we shift our gaze between these levels to capture the myriad ways that researchers perceive the relations between culture and mathematical knowledge.

## MATHEMATICAL KNOWLEDGE AND CULTURAL PRACTICE

These three levels allude to a conceptualization of math knowledge as inherently tied to cultural practices. This point has been highlighted in studies that examine the mathematical thinking and problem solving that students take part in outside of school. This body of research has highlighted both the complexity of mathematical thinking in everyday practices (even for unschooled people) and the ways that such knowledge transfers or fails to transfer into classrooms. As such, it speaks to issues of mathematics knowing as a cultural activity, or the first level of our model.

Researchers have studied a wide range of out-of-school mathematics practices, including shopping in grocery stores (Lave, Murtaugh, & de la Rocha, 1984), loading dairy cases (Scribner, 1983/1997, 1984), carpet laying (Masingila, 1994), money exchange (Brenner, 1998a; Guberman, 1996; Taylor, 2004), selling candy (Saxe, 1988, 1999) and other goods on the street (Carraher, Carraher, & Schliemann, 1985; Nunes, Schlieman, & Carraher, 1993), playing basketball (Nasir, 1996, 2000, 2002), dieting and measuring food portions (de la Rocha, 1985), farming in Brazil (de Abreu,

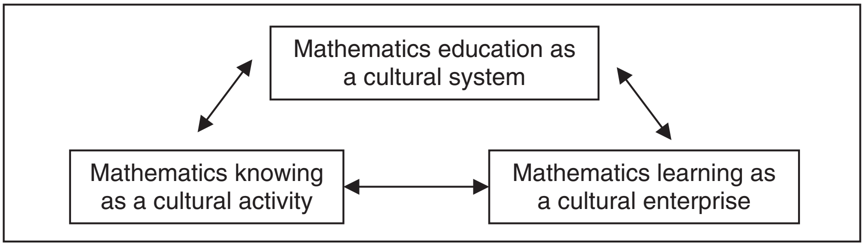


**FIGURE 1 Analytic Framework of the Boundaries Between Cultural and Mathematics Knowing**

1995), and math at work (Hall, 2000; Hall & Stevens, 1995; Hoyles, Noss, & Pozzi, 2001). There is also a body of research in ethnomathematics (Ascher, 1994; D'Ambrosio, 1985; Powell & Frankenstein, 1997) that highlights the various indigenous systems and practices of mathematics to problematize Eurocentric assumptions about valid mathematics and the power issues at play in deciding whose mathematics to legitimize. We do not undertake a review of research on ethnomathematics here, as our focus is on relations between domain and cultural knowledge for students in the United States. However, we begin with seminal cross-cultural research conducted in other countries that greatly informed this line of research.

One of the earliest studies in this area was a part of a broader effort to understand why Kpelle students in Liberia were underachieving in their Western-style mathematics classes. Cole, Gay, Glick, and Sharpe (1971) set out to better understand the kind of mathematics that the Kpelle encountered in their everyday lives (Cole et al., 1971; Gay & Cole, 1967. They documented extensive measurement practices (the Kpelle were rice farmers), including counting, classification, and the use of geometric knowledge for building houses. They designed studies that showed that the Kpelle did not use school-type approaches to solving these everyday mathematical situations but rather relied on visual and perceptual cues and specific cultural artifacts to estimate mathematical quantities. Similarly, Scribner (1983/1997), in a study of dairy case workers reported that although the workers were highly efficient in their solutions of on-the-job math problems, they solved such problems without the use of calculations, instead using routine visual displays. Lave et al. (1984) also explored everyday mathematics, examining the kinds of mathematical problems people solve as they go about their grocery shopping and also documented the difference between the nature of individuals' solutions to routine problems and approaches taught in school.

Similar findings have been reported for street vendors in Brazil (Carraher et al., 1985; Saxe, 1991, 1999) and African American high school basketball players (Nasir, 1996, 2000). Such studies have also found that when asked the same questions in an everyday and a school format, individuals tended to score higher on tasks more closely linked to their everyday practices, though schooled people do sometimes bring their school knowledge to bear to solve out-of-school problems. This line of research has illustrated that it is quite common for there to be strong boundaries in the minds of individuals (and in the practices themselves) between the kinds of local, practice-linked mathematical knowledge that people construct outside of school and school-linked mathematics. The extended example that opened this chapter is a part of this body of research. The players' response patterns are representative of similar patterns across other practices and clearly illustrate the boundaries between school math and out-of-school math for many youth.

Not only are there differences between school and out-of-school math knowledge, there is also wide variation within practices outside of school. In a study of the purchasing practices of elementary school African American students in an urban neighborhood, Taylor (2007) highlights important ways that the practice of purchasing outside of school is structured and scaffolded by others such that there is wide range of mathematical thinking that students engage in, depending on the difficulty of the problem that they are solving, the supports available to them, and potential time constraints for solving the problem. Furthermore, other research has explored the variation in solution strategies between school and out-of-school contexts and has highlighted the way in which solutions to math problems outside of school can draw on common sense, estimation, and cultural artifacts. For example, in one study, researchers documented that a woman faced with the problem of measuring three fourths of two thirds of a cup of cottage cheese simply filled a 1-cup measuring cup two thirds of the way full, dumped it onto an cutting board, and carved out one fourth of it, then put the remaining cottage cheese on a plate (Lave, 1988). This woman and others used nonconventional strategies rather than school-taught algorithms for solving mathematical problems in situ.

In addition to documenting the practice-linked nature of individuals' mathematical understandings and a widespread lack of usefulness of school-taught procedures in solving everyday mathematical problems, research has also highlighted the ways that math problems are solved in practices outside of school. Specifically, learners in everyday settings are often supported in various ways by social others as they attempt to solve authentic math problems in their everyday lives (Brenner, 1998a; Lave & Wenger, 1991; Nasir, 2000; Saxe, 1991, 1999). This support in the process of carrying out authentic tasks (formally termed an apprenticeship model of teaching) has been used as the basis for reforms in teaching practices in classrooms (J. S. Brown et al., 1989).

Another important finding from this work was that students who participated in the mathematics practices of their communities and who also attended school did not view both of these practices as having the same value or worth. Although students often used practices from their everyday or home math, they clearly felt that such practices were inferior and that the school math was of higher status or was more highly valorized by students (de Abreu, 1995; de Abreu & Kline, 2006). This issue of the way in which people make sense of the mathematical practices that they engage in and what those practices mean for who they are and how they fit into society brings to the fore issues of identity (Beach, 1995; Martin, 2000; Sfard & Cole, 2003). One way that these studies have been interpreted is that mathematics instruction should seek to better contextualize and make relevant to the real world its content (National Council for Teachers of Mathematics [NCTM], 2000). Sfard and Cole (2003) argue that these conclusions are a misreading of the findings from everyday math studies. Instead, they propose that these studies point to the importance of supporting the mathematical literacy of all students and that in the process, students' identities as math learners must be nurtured as well.

This body of research has supported a notion of math as inherently cultural activity, in part by pointing to how math knowing and learning looks different across different practices. In some ways, however, at the first level of our model, this set of studies has reified and made salient the boundary between cultural and domain knowledge. Yet at the second level, the studies also show how the teaching and learning practices in settings outside of school are constructed in ways that support novices in the development of cultural knowledge. Furthermore, these studies point to the constrained nature of school knowledge and problematize the privileging of school math knowledge. This body of research also provides an important lens through which to understand and study school mathematics classrooms, which are sometimes viewed as being acultural. Research on mathematical practices outside of school highlights the inherently cultural nature of mathematical activity and offers some insight into the types of cultural processes that are embedded in mathematical practices. Next, we consider three aspects of culture and math classrooms that have been prevalent in the research literature.

## CULTURE IN THE MATH CLASS

Considerations of the ways in which issues of culture show up in the math classroom are central to each of the three analytic planes. The math classroom is the local site through which the cultural system of math education is enacted, where particular types of math knowing are privileged over others, and where the cultural enterprise of math learning plays out in interactional space. In this section, we review research that has focused on the cultural nature of teaching and learning in math classrooms, highlighting three ways that research has considered culture in math classroom: (a) the way that language mediates knowledge, (b) features of math classrooms as contexts that support or constrain different forms of knowledge, and (c) the way that racialized identities and expectations play out in mathematics classes.

### Knowledge in Language

One important way that research has considered math learning as cultural activity is through an examination of the role of language in mathematics learning. How language in mathematics classrooms mediates meaning making and instructional practice (Cobb, Wood, & Yackel, 1993; Forman, 1996; Lemke, 1990; Lerman, 2001; Van Oers, 2001), as well as differential access for second-language learners (Brenner, 1994; R. Gutiérrez, 2002a; Khisty, 1995; Moschkovich, 1999, 2002; Warren, Rosebery, & Conant, 1994), has been the focus of considerable research during the past 15 years. Within these discussions, language has been conceptualized in myriad ways, relating to the nature of mathematical talk in the classroom, the discourse practices entailed in the learning of mathematics, and the challenges and opportunities of linguistically and culturally diverse mathematics classrooms.

First, and foremost, we follow theorists like Bakhtin (1981, 1986) and Vygotsky (1962), who considered language to be both a window into the meanings people make of themselves and their activities and the substance of these formulations. Language is a primary symbolic means through which we come to participate in and understand the world. In a sense, then, language constitutes and is constituted by knowledge. The understandings we develop as we enter into dialogue with social others (on the interpersonal plane) gradually become internalized (on the intrapersonal plane) and form the social and cultural fibers of our knowledge base (Vygotsky, 1962). It is this conception of language, as the primary source of our interaction with and reflection on the world (or how we know), that problematizes the ancillary position it is often relegated to with respect to knowledge (or what we know).

There is much to be said about the role of language in cognition and learning. Here, we will focus on aspects of it that are implicated in our analysis of the social and cultural aspects of knowing and doing mathematics. It is important to acknowledge that although language reveals and exposes certain meanings and interpretations, it necessarily obscures others. Like knowledge, it is never neutral. As our primary form of communication, however, language also allows us to achieve *intersubjectivity* (Lerman, 1996; Rommetveit, 1987; Schegloff, 1992), or the development of a shared understanding of the perspectives we bring to our activity together (Clark, 1996; Greeno, 2006b). These two characteristics of language—that it both hides and reveals meaning—are critically important to understanding the cultural processes of teaching and learning. As Erickson (2004) and many others have noted, the intricate linguistic acts involved in the coordination of meaning leave open the possibility for much misunderstanding and confusion.

The symbolic and abstract nature of the language of mathematics complicates the processes of communication even more (Durkin & Shire, 1991; Pimm, 1987). This complexity lies not only with mathematical syntax and register (or the terms, notations, and specific uses of them; Nemirovsky, DiMattia, Ribeiro, & Lara-Meloy, 2005) but also with the very structure of the practice of mathematics, which represents a distinct semiotic system (Lemke, 2002; O'Halloran, 2000, 2003). As an aspect of this semiotic system, language cannot be separated from other communicative practices such as gesture, alignment, and gaze, which function together to produce a discourse, or "the social activity of making meanings with language and other symbolic systems in some particular kind of situation or setting" (Lemke, 1995, p. 6). By positioning language as one of many components of a discourse, we limit the possibility of reducing complex interactions of mathematical activity to patterns in linguistic moves. We also understand why visual cues, artifacts, and social interaction play such a critical role in learning in everyday cultural practices.

Early work on discourse in the mathematics classroom tended to focus on how participants coordinated their activities through various discursive acts and how these emerged into patterns that came to characterize a particular type of classroom community. Researchers found that it was typically confined to a number of configurations. Historically prominent among these is the initiation–response–evaluation structure (Heath & McLaughlin, 1994), or IRE (Cazden, 1988; Mehan, 1979), which represents a closed system of meaning making in which the teacher poses a question, students attempt to respond to it, and this response is evaluated. This structure tends to constrain students' everyday mathematical register (Lemke, 1990), their mathematical conjectures (Wood, 1992), and the participation of lower socioeconomic groups of students (Heath, 1983). Despite this, the IRE structure continues to be pervasive in many "traditional" mathematics classrooms and limits the opportunities for classroom participants to engage in discourse that allows them to think together (Spillane & Zeuli, 1999).

Central to this analysis was the recognition that this coordination took place on multiple levels that comprised, for example, talking about mathematics and talking about talking about mathematics (Cobb et al., 1993). In other words, attention was paid not only to how particular mathematical meanings and conventions became regular features of the classroom conversation (both in whole-class discussions and small-group work) but also whether tacit assumptions about what it means to do mathematics were made explicit to students. In contrast to classrooms that used IRE structures, studies of discourse practices such as revoicing (O'Conner & Michaels, 1993), redirecting, probing, and such illustrated how expert teachers explicitly positioned students' mathematical utterances as meaningful with respect to the broader mathematics community, while at the same time clarifying what counts as a mathematical contribution (Ball & Bass, 2000; Boaler & Greeno, 2000; Cobb et al., 1993; Lampert, 2001; Rittenhouse, 1989).

Another line of research on classroom discourse also began to look more broadly at the relation between language, social practices, and power in shaping classroom life. Drawing on theorists in symbolic interactionism and sociolinguistics, such as Bernstein, Bahktin, and Goffman, researchers attempted to examine the multiple voices and historical artifacts within the mathematics classroom that stemmed from local communities and broader communities of practice (Forman & Ansell, 2001; Lerman, 2001; Van Oers, 2001). These accounts drew from cultural psychology and other anthropological traditions to consider the reflexive relations between the nature of mathematical conversations taking place in mathematics classrooms and the various positions from which different participants in the conversation are speaking. These studies found that teachers and students shifted between different discourse practices, depending on the goals of their activities, their cultural positions, and their alignment with different communities. For example, communicating social norms and exposing hidden characteristics of mathematical talk was marked by direct, explicit, and authoritative discursive moves (generally on the part of the teacher) to reshape conversation and, ultimately, the classroom culture. On the other hand, interaction regarding students' mathematical sense making was characterized by open-ended conversations and shared authority structures to foster students in making conjectures and taking risks (Cobb et al., 1993; Forman & Ansell, 2001). Whereas the latter focuses on negotiating meanings of mathematical ideas and procedures, the former helps students to understand why it makes sense to do this work within this community (Van Oers, 2001).

These findings led researchers to question the strong ties between mathematics classrooms and the mathematics community, suggesting that the two fundamentally differ in purpose and character (Moschkovich, 2000, 2002). In contrast to the mathematics community, a great deal of what goes on in a mathematics classroom is that students from different backgrounds are determining for themselves, in relation to the classroom community, what it looks like for someone like them to learn and do mathematics. However, as Van Oers (2001) argues, drawing on Bahktin, the *speech genre* of the mathematics community still predominates school mathematics and as such has significant sway over the look and feel of legitimate (and illegitimate) mathematical activity. We see this in the case of the basketball studies, where players described being able to express themselves through the practice of basketball while they either did or did not fit with the culture of the mathematics classroom (Nasir & Hand, in press). Lerman (2001) reiterates the importance of accounting for alignment and power in analyzing language in the mathematics classroom, arguing that (a) classroom discourse practices necessarily shape what is viewed as legitimate mathematical participation and (b) the official language of the classroom can position certain groups with power and privilege.

With respect to the first point, Wells and Arunz (2006) argue that the IRE structure limits classroom mathematical learning, as it assumes perfect intersubjectivity on behalf of the participants. They propose that classrooms need to be organized to foster dialogic inquiry, where the participants actively work to understand the speaker's perspective and attend to the speaker's focus of attention (Wells & Arunz, 2006). This perspective considers classroom learning to depend not only on the processes of acculturation—or learning the tools, meanings, and values of the mathematical community—but also on transformation, where the consideration of alternative explanations leads to new understandings. New reform mathematics practices support teachers in eliciting and building on students' ideas, thus opening up the possibility for dialogic learning experiences.

Even as mathematics researchers and reformers push for new and expanded discourse structures in mathematics, however, these initiatives do not necessarily conceptualize discourse as being embedded within critical social and cultural contexts, and indexing both local and broader discourses (Gee, 1990; Lemke, 1995; Lerman, 2001)—what our third level would push us to consider. The study of what Gee calls "Big D" discourse maintains the positionality of language within hierarchical power structures. It makes visible the various layers of meaning and relations that are indexed by and constituted in discursive acts. The discourse of school mathematics, then, can be traced to the activities and historical practices of particular communities that won decisive power struggles. In their work on hybrid discourse practices, *Third Space*, Kris Gutierrez and her colleagues argue that classrooms should be organized to circumvent or disrupt the societal power structures that leak into classrooms and constrain access to nondominant groups of students (K. D. Gutiérrez, Baquedano-López, Alvarez, & Chiu, 1999; K. D. Gutiérrez, Baquedano-López, & Tejada, 2000; K. D. Gutiérrez, Rymes, & Larson, 1995). One way to do this in classroom interaction is to position students' discourse practices as authorized ways of participating productively in the classroom. The hybrid discourse practices that result represent an expansive learning activity (Engeström, 2001), where participants reach new understandings by working toward common ground.

This is particularly important in classrooms with students from diverse linguistic and cultural backgrounds. As Moschkovich (2002) argues, students' mathematical sense making is grounded in their everyday discourse practices, which originates in the home and local communities. When the classroom linguistic structures are restricted to English, or teachers do not attend to the gestures, representations, and everyday descriptions that second-language learners draw on to create and communicate meaning, they inadvertently miss the multiple, rich resources that students bring to the classroom. The research of Rosebery, Warren, Conant, and others working with Haitian students in Chèche Konnen Center at TERC reinforces why it is critical to afford the development of the hybrid interactional spaces in the classroom. They have found that to create truly dialogic communities requires that teachers actively work to draw out and on students' resources for meaning through extended turns of talk and by puzzling out words and meanings that they can use to leverage students' disciplinary understandings (Rosebery, Warren, & Conant, 1992; Warren, Ballenger, Ogonowski, Rosebery, & Hudicourt-Barnes, 2001).

These perspectives illustrate the social and interdependent nature of the language, conventions, tools, values, and meanings of school mathematics. As a discourse practice, mathematics learning encompasses cultural ways of participating in mathematical activity that privilege (though not explicitly) particular ways of knowing and being in the mathematics classroom. Thus, developing mathematical and cultural intersubjectivity among students and teachers in ethnically, racially, and linguistically diverse classrooms depends not only on making an attempt to understand the meaning of a person's discursive act but also on creating space to renegotiate issues of power and status involved in this process.

## Features of Mathematics Classrooms

A second way that researchers have studied the relation between culture and math learning is by examining how opportunities to learn mathematics are structured in different ways within mathematics classrooms. For example, researchers concerned with design principles for mathematics classrooms explore how the norms and practices of mathematics classroom organize (and are organized by) different forms of agency (Chazan, 2000; Chazan & Ball, 1999; Engle, 2006), authority (Lampert, 1990), and accountability (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997) for students with respect to mathematical knowledge and the classroom community (Boaler, 2003; Cobb, Gresalfi, & Hodge, in press; Engle & Conant, 2002;

Greeno, 2006a; Kazemi & Stipek, 2001; Yackel & Cobb, 1996). However, the bulk of this work considers how these features support and constrain different forms of knowing and being for students in a general sense, instead of examining how they may be differentially available to particular groups of students. One of the reasons for this is that we have yet to develop an overarching analytic framework that captures how opportunities tied to classroom structures are shaped by processes that take place at multiple levels and time scales of classroom and social interaction.

One of the most careful and thorough studies of the teaching and learning of mathematics that addresses classroom structures at multiple levels of classroom interaction is Magdalene Lampert's (1990, 2001) practitioner-based research in her fifth-grade mathematics classroom. Lampert videotaped her classroom on a regular basis and kept detailed records of her practice, including her preclass preparation, on-the-fly decision making, and postclass reflections. In her analysis of these data, she examined how her students came to do mathematics over time by "zooming in" on particular classroom exchanges and "zooming out" on the patterns of activity that began to emerge over time. Through this process, she illustrates how classroom moments add up to the practices of a particular classroom community (in her case, modeled after the practices of mathematicians).

Yackel and Cobb's (1996) notion of sociomathematical norms helps us to further analyze how the practices of mathematicians became instantiated in Lampert's classroom. For example, part of what it meant to be a student in Lampert's classroom was to try out different mathematical ideas and to respectfully critique the ideas of other students, often in public. Through the processes of eliciting students' mathematical ideas and modeling appropriate questions to ask about these ideas, the students in Lampert's class became more practiced at constructing and responding to mathematical arguments in a form that mirrored the conversations of mathematicians. Thus, a sociomathematical norm emerged concerning "how one engages in a mathematical argument." Classroom exchanges in which this norm was violated were noticed by other participants and often repaired. This type of analysis contributes to our understanding of how the ritual ways of interacting around mathematical ideas, tools, and participants in a classroom evolve into a particular classroom culture (Franke, Kazemi, & Battey, 2007).

A line of research that has investigated the different types of cultures we find in mathematics classrooms has identified a number of key features of reform mathematics classrooms that foster students' productive, domain-based inquiry. These features include giving students the opportunity to problematize the subject matter in a way that is meaningful to them, distributing authority to them to develop and evaluate their mathematical methods, supporting students in exercising agency over the development of their mathematical understanding, and holding students' accountable to each other's mathematical thinking and questioning (Boaler, 1997, 2003; Boaler & Greeno, 2000; Chazan, 2000; Chazan & Ball, 1999; Cobb, Gresalfi, & Hodge, in press; Engle & Conant, 2002; Greeno, 2006a; Hiebert et al., 1997; Lampert, 1990, 2001; Yackel & Cobb, 1996). At the same time, researchers have also posed an important challenge to this research by questioning how what counts as an argument, method, or even mathematical activity is related to the forms of participation that students bring from home, local, and broader communities and discourses (Cobb & Hodge, 2002; Diversity in Mathematics Education [DiME], 2007; Moschkovich, 2002).

In the past 10 years, Jo Boaler and her colleagues have found that on the whole, reform-driven mathematics classrooms are more successful at narrowing the achievement gap than classrooms that focus on rote memorization and recall of procedures (Boaler, 1997, 2003, 2006a; Boaler & Staples, in press). She argues that in mathematics classrooms that use reform versus traditional mathematics curricula, students are more likely to be engaged in active problem posing and problem solving with the teacher and each other, where they are supported in thinking across ideas, methods, and formulas to build stronger and deeper connections to fundamental mathematical ideas. In her latest study of Railside High School, Boaler illustrates that the use of Complex Instruction (Cohen & Lotan, 1997), a multiple-ability treatment designed to rearrange status, in conjunction with reform mathematical practices can create multidimensional mathematics classrooms that broaden what it means to be "smart" in a mathematics classroom (Boaler, 2006b, 2006c). In Railside detracked math classrooms, students worked in groups on groupworthy mathematical tasks, which were structured by broader classrooms processes promoting explicit mathematical sense making and group accountability. Importantly, this study found that in the course of 5 years, students from diverse racial and ethnic backgrounds not only pursued higher-level mathematics courses throughout high school but also were more interested in the mathematics they were doing (Boaler & Staples, in press).

This research points to the growing recognition that it is crucial to locate issues of equity in mathematics education with curriculum and classroom structures rather than with individuals or groups of students. Multidimensional classrooms also challenge us to look beyond the a priori distinctions between mathematical and social activity we often make to how different forms of participation are framed and positioned around productive mathematical activity (D'Amato, 1996; Hand, 2003).

However, none of these accounts explicitly attend to how students negotiate the cultural practices they develop in communities outside of the classroom with those they encounter in the mathematics classroom. K. D. Gutiérrez and Rogoff (2003) have suggested that one way researchers might begin to explore these relations is by documenting students' repertoires of practice, which "characterize the commonalities of experience of people who share cultural background, without 'locating' the commonalities within individuals" (p. 21).

These studies on classroom mathematical structures illustrate how classrooms function as and in social cultural space to afford and constrain certain ways of doing mathematics and becoming a mathematics learner. This research has been increasingly concerned with the role of identity and how students come to see themselves as mathematics thinkers and doers.

## Racial Identities and Access

In a third category of research on culture in the mathematics classroom, scholars are exploring the ways in which students' racial identities and racialized opportunities and expectations have implications for their achievement in mathematics.

Ladson-Billings (1997) and others (Nasir & Cobb, 2006; Oakes et al., 2003; Secada, 1992; Silver, Smith, & Nelson, 1995; Tate, 1994, 1995) have pointed out the persistent achievement gaps in mathematics achievement by race. Although Ladson-Billings lays inferior instruction at the feet of these differences in achievement (citing Oakes, 1990, who shows that teachers of African American and other minority students are least likely to be prepared to teach mathematics),[2] she also suggests that one potential source of this disparity is the "nerdy" or "geeky" image of White males in horn-rimmed glasses that are conjured up in the public mind at the thought of high levels of mathematics achievement. In this section, we synthesize the emerging body of research on identities at the intersections of race and math learning as well as differential access (by race or ethnicity) to a wide range of schooling and mathematics learning resources.

In recent years, math education researchers have begun to explore not only the nature of students' experiences in math classrooms but also the extent to which students feel a sense of connection to math, or their mathematics identities. Boaler and colleagues (Boaler, 2002; Boaler & Greeno, 2000) have studied mathematical identities and math achievement for middle and high school students. Boaler's findings show that not all capable students in math have high mathematical identities and that the style of teaching had much to do with the types of mathematical identities students developed and their desires to continue to pursue mathematics (Boaler, 2002; Boaler & Greeno, 2000). Gresalfi and Cobb (2006) use the term *dispositions* to capture both the discipline of math as it is realized in particular classrooms and the extent to which students come to identify with the discipline. They recognize interest and connection as critical aspects of engaging in the practices of mathematics and draw on their own and others' work (Engle & Conant, 2002; Greeno & Hull, 2002; Greeno, Sommerfeld, & Wiebe, 2000) to focus on the way that students are positioned in interactions in math classes.

Martin (2000, 2006b) explores the way in which sociohistorical context and community norms come to influence students' mathematical identities through socialization processes. Martin argues that African American community members often identify their own limited mathematics achievement as being related to the constriction of opportunities to learn math because of racism and racial stratification. Students in mathematics classrooms must negotiate these historically rooted collective narratives about mathematics, which many students do with great success and assertion of individual agency. Martin argues that mathematical identities are constructed in relation to these sociohistorical forces as well as through local interactions and practices in schools and families. Furthermore, students are not mere pawns in their reactions to multiple conflicting narratives about math learning, race, and achievement—rather, they make agentic decisions that reflect their own ideas and goals.

Cobb and Hodge (2002) also employ the construct of mathematical identities to examine relations between culture and math learning. They theorize that power is an important factor in understanding identity and learning processes in mathematics. A number of researchers have conceptualized power relations within mathematics systems and classrooms in terms of classroom opposition (D'Amato, 1996; Diamondstone, 2002; Fordham & Ogbu, 1986; Hand, 2005; Stinson, 2006). In particular, Hand's (2005) research documented how opposition became a form of competent participation that grew among a highly diverse group of students in a low-track reform mathematics classroom. This opposition was related to the lack of opportunities for students to engage in mathematics, a discourse of tracking that positioned them as the "slow" and "dumb" students, and the teacher's resistance to a high-status discourse practice in the students' peer community that contained aspects of their mathematical sense making.

The studies cited thus far have primarily taken a qualitative approach to the study of race, culture, identity, and math learning. An important consideration is the extent to which these findings and issues define the experience of students on a broader scale. Although there is relatively little research on the relation between race or ethnicity, identity, and math learning or achievement, there is a large body of research on racial or ethnic identity and schooling outcomes more broadly. Research on the relation between racial or ethnic identities and academic achievement for African American and Latino students shows mixed findings (Burrow, Tubman, & Montgomery, 2006). Some studies find that students with stronger racial or ethnic identities achieve better (or have stronger academic identities; Sellers, Chavous, & Cooke, 1998; Supple, Ghazarian, Frabutt, Plunkett, & Sands, 2006; Thomas, Townsend, & Belgrave, 2003; Zarate, Bhimji, & Reese, 2005), whereas others report that students with stronger ethnic or racial identities perform worse (or have lower academic identities; Fordham & Ogbu, 1986; Steele & Aronson, 1995). Others argue that the effect of racial or ethnic identity on academic achievement is mediated by self-esteem (Lockett & Harrell, 2003).

One recent study has looked explicitly at the relationships between math identities, racial identities, and math achievement (Nasir, Atukpawu, O'Connor, & Davis, 2007). African American, Asian American, Latino, and White high school students were surveyed with respect to their mathematical identities, racial identities, academic identities, and math grades. Findings showed that although there were few variations in levels of students' racial or ethnic, mathematical, and academic identities by racial or ethnic group, African American students were less likely to connect their math grades to their sense of academic or mathematical identities than students from other groups and that particular versions of racial or ethnic identities were more supportive of achievement in mathematics than others.

Students' taking on or resisting identities as math learners do not occur in a vacuum. Although we will not do an extensive review here, we do want to acknowledge at least two other ways that culture comes to the fore in math classrooms. First, culture comes to play when teachers hold lower expectations for achievement and learning for students from particular racial or cultural communities (Ferguson, 2003). Studies on teacher expectations and race have highlighted the ways that differential treatment and expectations by teachers can greatly affect students' access to learning opportunities (Beady & Hansell, 1981). Second, issues of access can also constrict students' learning opportunities as well as opportunities to develop mathematical identities. More specifically, nondominant students, particularly, African American and Latino students and poor students, consistently have less access to a wide range of resources for learning mathematics, including qualified teachers, advanced courses, safe and functional schools, textbooks and materials, and a curriculum that reflects their experiences and communities (Apple, 1995; Darling-Hammond, 1997; King, 2005; Oakes et al., 2003).

Studies on racialized identities and expectations and their role in the teaching and learning of minority students in U.S. schools represents an important line of research. This work views the math classroom as both a space where students develop a sense of themselves as doers and learners of math and also where broader issues of power and access play out in fundamental ways.

Overall, research on the multiple ways that culture intersects with math learning highlights the myriad of ways that knowledge is inextricably linked to culture, language, identity, and power; situated in practice; distributed across individuals, tools, and forms of social activity; and structured by the features of social contexts that organize what constitutes knowing, how knowing is demonstrated, and how knowing is related to doing and being. In multiple ways, then, mathematics classrooms are inherently cultural spaces where different forms of knowing and being are validated. From the perspective of our model, the recognition of mathematics knowing as a cultural activity motivates a closer inspection of school mathematics learning as a particular type of cultural enterprise where these activities take place. This assertion counters the common assumption that mathematics knowledge is clear and precise and can be accessed by all and, thus, that mathematics teaching it is culturally neutral. However, as the research above indicates, what it means to know and understand mathematics, and what counts as productive activity toward knowing and understanding both in our classrooms and in society, is socially and culturally mediated.

This research on culture in the math classroom has spoken to all three levels of our analytic foci (see Figure 1). We have seen how knowing is defined locally (Level 1) and how configurations of activity in classrooms form cultural systems that students learn to navigate (Level 2). This body of work has also highlighted issues of access and power (Level 3), with an eye toward how these issues play out in local interactions in and out of mathematics classrooms.

## CULTURE AND MATH REFORM

Given this fundamental intertwining of culture and math learning, we must consider the ways that stakeholders in mathematics education have talked about and attended to issues of culture. In this section, we present a historical and critical examination of the treatment of culture within the context of reform mathematics and the challenges that have been faced in deeply considering culture and mathematical knowledge. We focus squarely on the third level of the model, or the cultural system of mathematics education in the United States. We argue that many of the challenges related to considering culture in mathematics classrooms are related to the historical motivations for mathematics reform and its resulting influence on the ways current mathematical goals and standards address issues related to the teaching of students.

## Historical Context of Considering Culture in Math Reform

The complex and situated nature of mathematics classrooms and mathematical thinking is made more complex by considering the relationship between the nature of student engagement and patterns of implementation of reform mathematics. The movement away from teacher-directed, top-down instruction and toward engaging students in meaningful problem-based instruction, as we have outlined in the previous sections, entails a greater consideration of aspects of students lived experiences. Tenets of reform result in increased communication, challenging of teachers and peers, and collaborative group work (NCTM, 2000), thus creating greater space for the influence of students' lived experiences and cultural communication patterns.

Underlying approaches to math reform is the belief that students learn best when they are asked to understand mathematics conceptually, not just apply formulas. This is certainly not a new idea. As early as 1899, the National Education Association (NEA) made calls for math instruction that demonstrated a strong interest in focusing on conceptual rather than procedural ideas. In its annual report, the group stated,

> While not wishing to undervalue models which are presented to the pupils ready-made, the committee believes that, as a rule, the pupils gain more by constructing their own models, and that this can de done very easily in a sufficient number of theorems. (NEA, 1899/1970, p. 203)

However, although it is a long-standing idea, math reformers did not begin to reconstruct approaches to teaching math for the masses until much later.

Interestingly, major changes in the need for math reform did not stem from a deep concern for issues of equity between students who are disenfranchised and those from the dominant culture or for a curriculum that was a more natural approach to teaching children. The reform movement grew out of an increasing concern for what seemed impending doom demonstrated by the perceived superiority of Russian scientific and mathematical capabilities as signaled by the launch of Sputnik (Schoenfeld, 2004). Central to this fear about international competition and losing the Cold War was a concern for the perceived need for mathematically and technologically skilled workers in industry and in the military.

These fears were summarized by the documents *A Nation at Risk* (Denning, 1983) and *Everybody Counts* (National Research Council [NRC], 1989), in which the authors call for serious change in the United States' commitment to education and raise concerns about the effects of the prevailing belief that mathematics education was a field of study reserved for the elite. These documents argued that the nation required greater numbers of students to achieve in mathematics education, attributed to the perceived deficiencies in the workforce caused by deficient skill in mathematics and science. It was posited that this workforce deficiency could weaken the nation's position in the world. According to the President's Commission on Excellence in Education, "America's position in the world may once have been reasonably secure with only a few exceptionally well-trained men and women. It is no longer" (Denning, 1983, p. 470). These documents were influential in bringing to the public not only the gap between U.S. math achievement and that of other countries but also the low number of students of color who were achieving in mathematics. They argued for the importance of an increase in the number of graduates prepared to take on roles in the fields of mathematics and education and that the low number of students of color in these fields contributed to this shortfall. Thus, although reducing the achievement gap was not the motivating goal of math reform, preparing a great number of the nation's citizens to be mathematically skilled was an important aspect of an agenda to maintain the strength of U.S. global power. Indeed, the italicized *all* in the emerging slogan "math for *all*" was pseudonymous for low-income students and students of color. Teaching "all" students resulted in a need to address the needs of students that were previously neglected in math education.

The concern for increasing achievement levels and learning in math and science also opened space for a reexamination of mathematics instruction. One argument suggested,

> Industry spends as much on remedial mathematics education for employees as is spent on mathematics education in schools, colleges, and universities. . . . This massive repetition is grossly inefficient, wasting resources that could be used better to improve rather that to repeat mathematics education. (NRC, 1989, p. 13)

Thus the neglect students in low-income and minority communities faced were not being addressed primarily from beliefs about the responsibility of government but rather as a cost-efficient solution for industry.[3]

Despite these initial motivations for addressing issues of equity and the consideration of culture, the impact from this "wake-up call" was twofold. First, it began to change the ways people perceived mathematics, and it also began to address beliefs about who can and should learn mathematics. Both of these goals would begin to shape the ways culture and race were viewed in relation to mathematics teaching and learning.

## From Crisis to Standards

Despite the shortcomings, *A Nation at Risk* and the calls from *Everybody Counts* led to a new set of standards for mathematics teaching that more carefully considered culture a significant role in curriculum and instruction in mathematics. Central to NCTM's concerns in the *Principles and Standards for School Mathematics* (NCTM, 2000) is the equity principle, which simply states, "Excellence in mathematics education requires equity—high expectations and strong support for all students" (p. 11). Equity and the idea that math reform should serve all students appear in multiple places in the NCTM standards document as well. Consider the following excerpts:

- "Because students learn by connecting new ides to prior knowledge, teachers must understand what their students already know." (p. 18)

- "Expectations must be raised—mathematics can and must be learned by all students." (p. 13)
- "Teachers need help to understand the strengths and needs of students who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics. . . . They can then design experiences and lessons that respond to, and build on this knowledge." (p. 14)
- "All students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests." (p. 13)

Creating success for all meant that addressing culture in the math class got operationalized as understanding what individual students already know and raising expectations for a wide range of "diverse" students. However, with this acknowledgement, the document places race and culture with language and disability as challenges to teaching rather than a central consideration in mathematics instruction.

General calls for higher standards and better math instruction for all students does not, however, address the root of inequality, nor does it acknowledge the social realities in schools for many marginalized communities and students (Martin, 2007). Apple (1992) cautions against such calls for high standards and emphasis on technology without considering the existing social and political context that influence how and what students can learn. He states, "Originating motives *do not guarantee at all* how arguments will be used, whose interests they will ultimately serve, and what the patterns of differential benefits will be, giving existing relations of unequal power in society" (p. 438). Indeed, this may only exacerbate the difference in the quality of education received by those who have been traditionally disenfranchised. Apple calls such approaches a "slogan system" that provides challenges to issues of equity. He asks whether it is appropriate to write standards under the assumption that there is equality between communities in their access to technology when communities are, in fact, demonstrably unequal. Tate (1994) goes further to state,

> The federal government's position on mathematics standards is akin to the concept of a toll road. In order to benefit from a toll road, the driver must be able to afford the cost of driving on it. Similarly, those school districts that cannot pay the long-term costs of implementing the new mathematics standards will have students who do not benefit. (p. 387)

Others simply argue that the math being taught has such little relationship to mathematics used in industry that the current system does much to add to inequities in education (Noddings, 1994).

Although we have alluded to some of the pedagogical differences between traditional and reform curricula, we should also highlight some fundamental differences in beliefs between supporters of each of these types of curricula in defining what constitutes mathematics and the purpose it should serve for individuals and society. Many of these differences are highlighted in the book *The Saber Tooth Curriculum*

(Benjamin, 1939), a satirical account of a "caveman" society where elders try to determine what children in their prehistoric community should be taught. In the book, New-Fist asks, "What things must we tribesman know how to do in order to live with full bellies, warm backs, and minds free from fear?" (p. 28) Emblematic of this, the New-Fist curriculum was full of important knowledge such as fire scaring and horse clubbing. When these animals no longer roamed their lands, an argument ensued between elders in determining whether to continue to teach these now-obsolete skills because they were fundamental to all learning or to teach new hunting and fishing skills relevant to the present day. This book, written by an education professor under a pseudonym, parodies the arguments of academics and politicians of the time who stood by what he saw as an outdated system of education. These questions continue to mark a sharp contrast in math education today. Is math to be taught as skills that develop "discipline of mind" or an aesthetic understanding of a cultural invention, or rather, should mathematics be a continuously developing tool based on logic and problem solving that can be used effectively in society?

## Equity Promises and Challenges of Reform Math

In addition to putting forth a vision of the goals for mathematics teaching, the standards also outlined a set of practices and approaches that constituted "reform" mathematics. A standard traditional problem that asks students to find the mean, for example, by following the procedure of summing amounts and dividing by the number of items summed looks very different than a corresponding reform task where students are asked to collect data from fellow students, compile data, and make arguments about which measure of central tendency is most appropriate for the particular data set.

A major tenet of reform mathematics is problem-based instruction, where students make conjectures and reasoning about particular mathematical ideas. As compared to traditional mathematics with often one solution and where the teacher or textbook serves as the ultimate authority as to right and wrong solutions, reform math asks students to serve as their own authority, to make mathematical arguments. These shifts in both problem-solving process and the distribution of authority may have implications for the participation of low-income and/or minority students.

Researchers have described both affordances and constraints for reform approaches' consequences for equity and the learning of nondominant students. For instance, as mentioned earlier, Boaler's (1997, 2006b) longitudinal research on mathematics education in the United States and the United Kingdom in the past 10 years has identified positive links between the classroom features of reform math and learning outcomes. Building on this work, Horn's (2006) research illustrates how reform mathematics curricula also afford development of a different set of categories with which teachers can begin to challenge deficit perspectives of their students. She found that in contrast to assigning labels such as *fast*, *slow*, or *lazy* to children who struggled under a curriculum that emphasized competition, speed, and the memorization of procedures, teachers who taught from a reform curriculum were concerned with developing ways to elicit their students' mathematical understanding and sought out techniques and strategies from colleagues to help build on the knowledge that they knew their students possessed.

Communication is central to reform mathematics learning. Yet these approaches often privilege a type of communication that is more prevalent in particular communities. Lubienski (2002), for example, found that although students from higher socioeconomic-status families expressed confidence in expressing their ideas during classroom discussions, stating, "I want other people to understand my ideas," lower-income students were more likely to report that they did not participate in classroom discussions for reasons related to confidence, such as "I don't like to be wrong in front of the whole group." Similarly, these students' small-group discussions were qualitatively different from that of the higher-income students; the form of communication by these students allowed for discussions of the work that did not necessarily include the deeper mathematical ideas that the lessons were designed to support.

Norms that privilege particular socioeconomic groups or ethnic groups have also been demonstrated with respect to cultural expectations of peer and teacher interaction. The idea that it is appropriate to argue with peers or with teachers has been a challenge for some students as they engage in reform classroom discourse (Murrell, 1994). How does one address the perceived conflict between respecting elders and the expectation to challenge teachers' explanations and solutions to mathematical problems? How do students balance this expectation with their own cultural norms?

## Challenges to Equitable Implementation

In addition to these potential challenges inside of reform math classrooms, there are also substantial challenges to the implementation of reform in schools that serve students from nondominant and poor communities. An examination of National Assessment of Educational Progress data demonstrated that schools with students of color were more likely to adopt more traditional practices, such as multiple-choice assessments, as compared with White students who received greater instruction using reasoning in solutions of novel problems (Strutchens & Silver, 2000).

This unbalanced relationship between ethnicity and class with the implementation of reform appears to be threefold. First, students who attend low-performing schools were most likely to receive a back-to-basics curriculum (Oakes, 1990; Oakes et al., 2003), often to improve state-mandated test scores. Low-performing schools have been consistently associated with a low-income and minority student body. Second, studies suggest that a reform curriculum requires greater pedagogical knowledge and preparation by teachers to ensure optimal learning outcomes. Teachers in these disenfranchised districts more often have less content and pedagogy training to effectively teach using these new curricula (Darling-Hammond & Sykes, 2003). This lack of training greatly decreases the value of reform curricula. Research has found little difference in students' learning of mathematics concepts in classrooms where teachers used a reform curriculum paired with traditional teaching techniques as compared to those where teachers used a traditional curriculum alone. Only in classrooms where teachers used both reform pedagogy and a reform curriculum did students demonstrate more advanced mathematical thinking (Saxe, Taylor, McIntosh, & Gearhart, 2005).

These challenges to the implementation of reform, taken together, create a cycle of underachievement where students in low-performing schools experience back-to-basics instruction and have little exposure to larger conceptual issues highlighted in reform approaches. Because of a lack of success with reform curricula attributed to poor implementation, training, and incompatible beliefs about student behaviors, these schools are likely to change to traditional curricula or never adopt reform curricula initially.

Third, social and sociomathematical norms in many of these schools may be in conflict with the norms and expectations of reform mathematics. Ladson-Billings (1997) states that in many urban communities, the definition of "good school" is one that maintains high levels of order and control. This privileging of order and control are seen both in goals set by urban districts in regard to zero-tolerance policies (Casella, 2003) and in media portrayals of strong-handed principals such as "Crazy Joe" of the movie *Lean on Me* (Twain & Avildsen, 1989). This privileging of order and control may be in conflict with tenets of reform that include the need to question, to argue, and to explore.

A relatively new influence on the way a reform curriculum is implemented and the quality of that instruction in classrooms that serve students of color is the NCLB of 2001. This influence is related to requirements of NCLB with respect to the hiring of teachers, allocation of funds, and particular forms of progress standards used by individual states to measure the adequate yearly progress (AYP) of all students. But the built-in flexibility of the federal policy may result in widely differing effects on student achievement because of the different ways states and districts implement particular portions of the act (Jones, 2006).

As we have noted, much of the challenge of successful implementation and sustained support for reform mathematics implementation may be linked to teachers' discomfort with reform curricula and competency with deep mathematical ideas. Thus in districts with lower-income students, successful implementation may be directly related to the increase in licensed teachers or those holding majors in mathematics. As a focal issue, NCLB called for all teachers to be "highly qualified" in the subject they teach by the end of the 2006–2007 academic year. This call was a reaction to findings that demonstrated wide differences in the mathematical preparation of teachers from various districts and states as well as the generally low numbers of math teachers with mathematics degrees in the United States as compared to other industrialized nations. In the year 2000, whereas 90% of middle and high school math teachers in Minnesota held majors in mathematics, states such as California and Tennessee had dramatically lower rates, 57% and 54%, respectively (Erpenbach, Forte-Fast, & Potts, 2003). Though data are still being collected to determine the extent to which NCLB has made substantial changes, the trend has certainly moved to greater licensing of teachers and greater numbers with mathematics degrees in even the most impoverished communities. Although these changes may not guarantee an increase in the implementation of any particular type of curriculum, it is likely that high-quality teachers are better able to implement the more difficult mathematical tasks presented in a reform curriculum, if indeed this measure represents better mathematical understanding.

Another aspect of NCLB that may also influence this implementation in many low-income and minority communities is the focus on standardized annual testing. Indeed, this requirement brings to the fore chronic academic neglect in some districts, but its effect on curriculum implementation and instruction is dependent on the way that states define progress and, indeed, learning. Because NCLB allows for states to determine an appropriate level of progress and, indeed, sufficient mathematical knowledge, much of the influence on reform implementation will be related to goals and proficiency levels decided on by individual states. Assessments that focus specifically on basic skills, or broader assessments that include more difficult problem-solving tasks but also set the "passing" benchmark at a basic skill level, leave higher-order mathematical problem solving unmeasured or merely as superfluous advanced knowledge. It is these higher-order skills that align more closely with reform mathematics curricula; thus if basic skills are used to determine success, then traditional methods and basic goals may be reinforced, ignoring benefits of reform curricula. What remains to be understood is the way that particular communities have measured success as a function of class, race, and language.

The equitable distribution of opportunities to learn powerful mathematics is clearly one of most pressing issues in the multiple gaps in mathematics education that exist at the intersection of cultural and domain knowledge (Moses & Cobb, 2001; Schoenfeld, 2002). The history of mathematics reform indicates that this issue is bound up in communities with more or less power; political and social systems that perpetuate systemic poverty, injustice, and privilege; and "folk" discourses about differential achievement of various groups of students, which together implicate mathematics learning as a cultural system. The fact that this cultural system has historically reinforced a narrow vision of what it looks like to learn and become good at mathematics suggests that implementing reform mathematical practices with fidelity into more low-income classrooms with predominantly students of color may not be adequate (DiME, 2007; Martin, 2007).

## BLURRING THE LINE BETWEEN DOMAIN KNOWLEDGE AND EVERYDAY KNOWLEDGE AS A SOURCE OF EMPOWERMENT

In the prior section, we considered the role of culture in both the history and practices of reform math. We noted that some teachers have demonstrated that reform classrooms can be optimal learning spaces for a wide range of students. Other researchers and educators have developed teaching approaches specifically geared toward teaching and empowering nondominant students, families, and communities. One approach has been to leverage students' everyday social and cultural knowledge to improve domain-related understanding. These approaches seek to make use of the rich sources of knowledge that exist outside of the classroom in the varied activities of cultural life to improve students' participation in classroom activity. In doing so, they attempt to blur the line between domain versus everyday knowing and learning and often work at all three levels of our analytic model. Such approaches also take seriously issues of race, academic identities, and access. Consequentially, students are potentially afforded opportunities to gain increased authority to participate in mathematics in ways that honor and validate their everyday identities and practices.

In this section, we consider several specific programs of research and approaches to teaching that seek to challenge the boundaries between domain understandings and cultural knowledge. These approaches draw on the everyday cultural understandings of students to support domain knowledge in mathematics. In addition to making an explicit link between the everyday and the academic, these programs position their work as challenging existing hegemony in educational spaces, opening up such spaces as sites for inquiry, and repositioning "minority" students with respect to the knowledge they produce in mathematics class. Thus, an important aspect of these projects is to challenge positions of power and privilege that are reinforced in traditional separations of domain and cultural knowledge.

We discuss and offer examples of programs that have attempted this "blurring," including the Funds of Knowledge Algebra Project, social justice approaches to mathematics, and culturally relevant pedagogy. Clearly, because of space constraints, this list is not exhaustive (other important examples of this work include that of Lipka, 1994, 2005; Lipka & Mohatt, 1998; and Brenner, 1998b). Rather, we use these programs as a way to illustrate the themes that are present in this approach. For each, we will briefly describe the research and teaching approach, highlighting the core aspects of the program, and its stated goals. We also explore important themes that cut across these various approaches and consider challenges to these approaches.

## Funds of Knowledge

The Funds of Knowledge approach (Civil, 2002; Gonzalez, Andrade, & Carson, 2001; Gonzalez et al., 1993; Gonzalez & Moll, 2002; Moll, Amanti, Neff, & Gonzalez, 1992) takes as a central the idea that cultural communities have strengths and important knowledge bases that currently are not tapped in schools. Their work focuses on Latino and Native American families and communities and documents the wide range of important skills and knowledge bases parents, students, and other community members hold that could be viewed as a resource to teachers and schools. Their work has focused on three different aspects of the relation between parents' and community funds of knowledge and schools. Early phases of the project involved trained ethnographers conducting extended visits to the homes and families of students and documenting the extensive community and family cultural practices that students and parents engaged in as a part of their daily lives. Ethnographers found that parents and youth were involved in a wide range of important and domain-relevant practices, including folk medicine and animal husbandry, construction, sewing, mining, religion, and appliance and automobile repairs. Findings from these studies were shared with teachers, and teachers were asked to consider how understanding families in these new ways might change their teaching practices.

A second aspect of this project involved establishing teacher–researcher working groups, in which teachers were supported in conducting their own ethnographic studies of the home lives of a few of the students in their classrooms. Researchers and teachers met regularly to discuss observations and brainstorm ideas about how to better incorporate the findings from the ethnographic investigations into classroom instruction as well as to debrief the process of the conducting of the research itself. Teachers' involvement with their students' home lives shifted their perceptions of students and their capabilities drastically (in many cases) and offered teachers a better sense of the "whole child" in their classrooms as opposed to just the part of the child that showed up in the life of the classroom.

A third aspect of the project built on the other two and involved incorporating parents and community members directly in classroom instruction as experts. In one example, a parent drew on her skills at making candy and did a lesson with the students on making Mexican candy. This lesson led to a discussion with students on variations in the economies and buying and selling practices in the United States and Mexico.

The Funds of Knowledge approach bridged cultural and domain knowledge by connecting families, communities, and schools. This bridging process involved building multiple kinds of connections, from community practices outside of school to school practices. An important component of the connection between community and school knowledge was content knowledge about mathematics and other domains, such as science and literacy. In other words, students' experiences outside of school—with gardening and construction, for example—were used by classroom teachers to better support students' learning of important mathematical concepts (Civil, 2002).

Another type of bridging that the project supported was a bridging of power. In other words, the project shifted the normal asymmetrical power relations between families, teachers, and schools by repositioning families and students as smart and capable and as having knowledge that is valuable to the whole school community. The project fostered relations between parents and teachers, where the parents took on the role of the expert and teachers took on roles of learners and facilitators. In essence, this project recognizes that typically, certain kinds of knowledge are privileged in schools and that "school" knowledge tends to hold power and position those who have it as powerful, whereas everyday knowledge tends not to hold the same level of status and power. One important aspect of this work, then, is the ways in which power is reconstructed when teachers come into parents' homes as learners and when parents come into classrooms as teachers.

## The Algebra Project

The Algebra Project was started by Robert Moses, a well-known civil rights organizer, to support the learning of algebra for African American and other disenfranchised students who may not have access to learning higher-level mathematics (Davis et al., 2006; Moses & Cobb, 2001). Moses views mathematics as an important civil rights issue, because having access to strong algebra instruction in middle school can prepare students to complete higher levels of math in high school and thus to be competitive for college and gain access to important technological fields of study at advanced levels. Underlying this focus are both long-standing racial gaps in mathematical achievement and course-taking patterns and the traditional separation of formal mathematics from the experience of young people.

Algebra Project instruction differs in fundamental ways from textbook approaches to the teaching of algebra. Instead, the curriculum takes a project-based approach, grounding the solution of algebra problems in the real-life experiences of students, then asking them to reflect mathematically on those experiences. Moses and Cobb (2001) write,

> We are using a version of experiential learning; it starts where the children are, experiences that they share. We get them to reflect on these drawing on their common culture, then to form abstract conceptualizations out of their reflection, then to apply the abstract back to their experience. . . . Each step is designed to help students bridge the transition from real-life to mathematical language and operations. (pp. 119–120)

Moses and Cobb (2001) outline five steps to the Algebra Project curriculum process: (a) physical events, where students share a physical real experience in the world; (b) pictorial representation or modeling, where students find a way to represent that experience on paper; (c) intuitive language or "people talk," where students discuss and write about the physical event in their own language; (d) structured language or "feature talk," where students make use of structured language for the purposes of selecting and encoding features of the even that are relevant for further study; and, finally, (e) symbolic representation, where students construct symbols to represent their mathematical ideas. One important physical event occurs as a part of the mathematics-of-trips unit, where students take a physical trip on a bus or subway and then reflect mathematically on various aspects of the trip.

Further dissemination and supporting the developmental trajectories of young people in mathematics are important aspects of the Algebra Project. Dissemination and making the project available to a wide range of youth in communities across the nation come as a product of extensive teacher-training programs and the will on the part of the project to argue for reflective teaching in a standards-based educational climate. Youth are supported beyond their experiences as middle school algebra students through the young people's project (YPP) for late adolescents and young adults, which offers young people a continued connection to other young people pursing the learning of mathematics and also provides mentorship and tutoring for the younger students.

Funds of Knowledge and the Algebra Project illustrate pedagogical approaches that leverage different forms of knowing and build on strengths and funds of knowledge that each student brings to his or her learning to create a richer and more inclusive learning environment. In doing so, they work at all three levels of the model simultaneously, creating spaces where cultural knowledge has an important place in the math classroom, where the cultural system is fundamentally inclusive and encourages participants in the larger mathematical enterprise. However, researchers concerned with issues of power and race may argue that this is not enough. Instead, they contend that schools must provide students with the knowledge and tools to act back on those structures that currently (and historically) serve to disenfranchise them and their communities. It is not simply a matter of giving students greater access to what has been labeled by some as "dominant" mathematics knowledge (R. Gutiérrez, 2002b; Gutstein, 2006), or knowledge that is fixed within the culture of power and perpetuates the existing social hierarchies. As we have argued, knowledge (or a knowledge system) is never neutral and as such is necessarily linked to power in the way that it privileges and marginalizes certain perspectives and narratives. This aspect of knowledge, as a source of power, has traditionally been overlooked in research that examines the relations between knowledge, culture, and differential achievement.

## Social Justice Curriculum

A social justice approach to mathematics teaching raises issues about the purposes (political and social as well as cognitive) for which we engage in or ask students to engage in the study of mathematics. Such curricula ask teachers and students to employ mathematics as a tool to critically analyze and act on inequitable situations in their communities. Frankenstein (1983) cites several reasons for taking this approach toward mathematics education. The first is to expose the myth that (mathematics) knowledge is a value- and culture-free product of an objective and rational process of deduction. Exploring the tradeoffs we make in the process of using particular forms of mathematics to capture and represent various social phenomena, for example, positions mathematics as a tool for cultural and political purposes.

A second reason is to challenge what Freire (1970) called "massified" consciousness, where individuals who are oppressed take part in their oppression by believing that they operate independently, out of free choice, instead of within broader social and cultural systems. Citing Apple (1979) and others, Frankenstein (1983) describes how prevailing ideologies and categories that frame what it means to be a mathematics knower (and thus an elite member of the technical ruling class) perpetuate the belief that effort is unrelated to mathematical competence and that failure is an individual consequence. Critical literacies (and in this case, critical mathematical literacy) with special emphasis placed on unpacking language and nurturing conflicting views foster the development of *conscientização*, or "critical consciousness," which can motivate individuals to challenge a system of mathematics that leaves them with less power.

The third reason for a social justice orientation in mathematics education is, to Freire, the most important to realize and, in practice, perhaps the most difficult to achieve—to directly involve students in social activism. According to Freire (1970), it is not enough to simply present students with mathematical problems that authority figures (such as the teacher) view as unjust or unfair. (This serves only to deepen the massified orientation.) To emancipate themselves from the tyranny of their own oppressed thinking, students must be given the opportunity to determine which issues are most relevant to them in their schooling and how mathematics can be used to address these.

The emphasis on action is deliberate. Social justice theorists contend that granting students access to more and higher quality mathematics education only continues to foster their complicit participation in a system that disenfranchises them. At the same time, however, Frankenstein and others have acknowledged the realities of teaching mathematics for social justice in today's mathematics classrooms. We discuss some of these issues in the sections that follow.

Gutstein (2003, 2006) navigated these tensions directly in his work as a middle school mathematics teacher teaching for social justice in an urban Latino school. Drawing on Freire's critical consciousness and problem-posing pedagogy, as well as the notion of positive sociocultural identities in Ladson-Billing's work on culturally relevant pedagogy, Gutstein presents a framework that maps the components of a pedagogy for social justice directly to mathematical goals. The three social justice pedagogical goals include (a) reading the world with mathematics, (b) writing the world with mathematics, and (c) developing positive cultural and social identities (Gutstein, 2006, p. 23). These correspond directly to three mathematical pedagogical goals that include (a) reading the mathematical word, (b) succeeding academically in the traditional sense, and (c) changing one's orientation to mathematics. Following Freire's claim that reading and writing the world are dialectical, Gutstein (2006) emphasizes that the social justice and mathematics goals function interdependently and are mutually reinforcing, which he documented through his students' reflections and mathematics work.

In the course of 2 years, Gutstein observed his students using mathematics to pose and solve mathematics problems that allowed them to critique the material and social conditions of their lives (including his knowledge as their teacher). For example, when exploring the unequal distribution of wealth among nations, students questioned whether one could truly assess which country had more wealth without knowing about how these resources were distributed within each country. In another project, students constructed complex arguments about why racism may or may not play a role in median housing prices across neighborhoods and how they would know for certain. In each case, mathematics was positioned as a tool to make sense of the world at the same time as it was situated within and for particular purposes. The mathematical findings that students discovered also led them to generate new questions about hidden patterns and relationships that they deemed relevant.

After seeing that mathematics was within their reach and had a direct impact on their lives, a majority of Gutstein's students also developed a stronger relationship with mathematics. Not all of them liked mathematics, but most recognized its role and power in shaping their world. This is echoed by one of the students when asked how much her views had changed from being in the class:

> A lot. Two years ago I didn't really care at all. I've just noticed that since the past two years, I've been more interested in the world, and the ways things are (in terms of wealth distribution and population). I've been watching the news ever since. (as cited in Gutstein, 2006, p. 63)

## Culturally Relevant Pedagogy

Demands to consider the substandard education of students of color as a political imperative is echoed in the work of Gloria Ladson-Billings (1994, 1995) on *culturally* *relevant pedagogy*. Speaking to the disheartening findings that desegregation appeared to be favoring White students at the expense of African American students, Ladson-Billings provoked the field to take seriously the concerns of educating African American children as a unique population of students. Culturally relevant pedagogy represented a significant move away from superficial and essentialist versions of multicultural education epitomized by celebrations of ethnic foods and holidays toward an awareness of the different ways of communicating and being that African American students brought to the classroom (Ladson-Billings, 1994, 1995; Tate, 1995). Ladson-Billings asserts that teachers who taught the color-blind perspective that all students should be treated the same tend to perceive nondominant students constantly in relation to (and inferior to) their White peers. The pervasiveness of a deficit perspective of African American children also led to a missionary approach to teaching, where teachers entered the profession to "save" underachieving students of color from dropping out of school, instead of holding them to high standards for their academic work.

Examining the work of teachers who are effective at both holding their African American students to high standards and ensuring their place in the classroom community, Ladson-Billings (1994) has developed a set of definitions and indicators for culturally relevant pedagogical practice. This level of detail of teacher practice is situated below three overarching components, including academic achievement, cultural competence, and sociopolitical consciousness. By *academic achievement*, she means the capacity of teachers to effectively articulate and meet multifaceted and individualized goals that correspond to the needs of each student. By *cultural competence*, she means the propensity of teachers to spot, consider, and capitalize on the cultural practices and sensibilities of their students. By *sociopolitical consciousness*, she means the orientation that teachers have on social issues such as racism, social justice, and privilege and the ways they relate content to context. Broadly speaking, these components are less about the teaching of particular subjects and more about the stance that teachers' take toward the relation between school, cultural, and political knowledge. In other words, the aim of culturally relevant pedagogy is to explore the nexus of school, home, community, and society in the context of African American achievement.

Themes in culturally relevant pedagogy pervade much of the research we have described above (Gutstein, Lipman, Hernandez, & de los Reyes, 1997) and theories for teaching diverse learners (Banks et al., 2005; Foster, 1995). Foremost among these is the recognition that there is no single knowledge base that encompasses what teachers need to know to foster culturally responsive and academically rigorous classrooms. Instead, these theories consider knowledge for teaching to be embedded in teacher practice and continually evolving through teachers' reflections on their interactions with their students, their students' communities, peers, and others.

Similar to the Algebra Project and Funds of Knowledge, both social justice approaches and culturally relevant pedagogy integrate all three levels of the model in their approaches to teaching (see Figure 1). These approaches integrate a concern with mastery of knowledge and broadening what counts as knowledge, shifting norms of and discourse about important mathematical problems, and repositioning students to extend greater access and power over their mathematical experiences.

## Cross-Cutting Themes and Challenges

Several themes cut across the approaches we have reviewed to blurring the lines between cultural and domain knowledge in mathematics teaching and learning. First, there is an acknowledgement that mathematics teaching and learning are fundamentally cultural activities that to date have privileged certain students and communities rather than others. Second, (and it follows, that) all of these programs challenge traditional assumptions that some students are more able to learn than others (and too often these two categories correspond to racial and social class groups). Last, all of the approaches we have described make the assumption that all learners are capable, given the appropriate support, challenge, and instruction.

These approaches also take as central the task to connect students' experiences in the mathematics classroom to their experiences in everyday life as well as to political and social issues relevant to their lives (but that they may not have considered). Thus math teaching is both about building on what students are familiar with, so as not to alienate them, but it is also about introducing new ideas, concepts, and sensibilities. This involves a complex process of validating students' current identity and sense of themselves while expanding them to include new kinds of social, political, and mathematical activity. In doing so, these models of teaching and learning view math not simply as cognitive activity but also as social and political activity—activities that we do with one another as we seek to improve our world and push for social justice. In this way, the teaching and learning of mathematics becomes a vehicle for shifting current power relations, to use mathematics for the purposes of empowerment at both the individual and community levels.

However, these approaches are not without challenges. We discuss three of these challenges: (a) the difficulty of keeping the math in full view when building on everyday knowledge or when talking about social justice; (b) implications for ethnically heterogeneous classrooms, where students may not share cultural background or community experiences; and (c) the mismatch between these approaches and the teacher workforce and structure of the profession.

The first challenge that has been identified particularly in the work of Funds of Knowledge and teaching math for social justice is the difficulty of keeping the math in view while deeply inquiring into the everyday social world. Activities such as constructing gardens (Civil, 2001) and analyzing world distributions of resources (Gutstein, 2006), for example, are necessarily social and cultural phenomena in which mathematical forms and expressions can be oversimplified (e.g., calculating averages) or extremely complicated (e.g., high-level statistical modeling). It is also the case that students' mathematical moves in these activities may be quite varied and unconventional and thus pose a challenge for teachers to build on productively. In the Funds of Knowledge work, Civil (2001) has found that mathematical dilemmas often arise naturally out of students' everyday activities but that it is important for the teacher to be able to shift students' focus from the concreteness of the everyday situation to abstract principles and procedures in mathematics.

The research on the implementation of social justice tasks in the mathematics classroom has also prompted questions about the difficulty of balancing discussions of complex social issues with the mathematics. For example, Bartell (2006) reports that teachers in her professional development course on social justice in mathematics found it challenging to move flexibly between the mathematics content and conversations about social injustice. Gutstein (2006) also admits that in his class, he had to occasionally forgo opportunities to pursue mathematical investigations to deepen the conversations about social issues. Because students often hold strong perspectives about social injustice and these can trigger emotional responses, it is clearly important for teachers to be able to adeptly and sensitively guide students back to the mathematics at hand. We wonder, then, if teachers are being prepared and have bargained for doing this multifaceted work. Also, can social justice activities stemming from students' social realities sufficiently drive students' development of sophisticated mathematics knowing across an extended period of time? Or should they mainly serve as supplemental materials to the existing mathematics curriculum, used to convince students that learning mathematics is relevant and even critical to improving their lives and the lives of others?

A second challenge is in thinking about how one might apply these approaches in racially or ethnically heterogeneous classrooms. This is an especially salient issue for Funds of Knowledge and the Algebra Project, as to some degree, these approaches assume a degree of coherence within the communities that are being served. How might these approaches be adapted in classrooms where students are from multiple communities? What might it mean to draw on students' experiences in such multicultural classrooms? This may be less of an issue with social justice approaches, but it still leaves more to be negotiated with and between students in classrooms where there is a wide range of race, class, or socioeconomic groups represented. How are teachers to deal with kids from communities that do not share a social justice perspective or see social justice in terms of their own philanthropy? Would such students (not from working-class families) buy into the basic premises of this approach? What additional support might they need to do so? Another critical issue with social justice approaches is that the time spent on social justice issues is potentially time not spent on math. What about the middle-class and upper-class parents who are unwilling to sacrifice time for "basic math" and relegate these approaches as appropriate only for those from nondominant groups? Similarly, in considering heterogeneity and culturally relevant pedagogy, it may be more difficult in heterogeneous classrooms and communities to have a sense of the community that students come from; there may be greater differences in achievement and histories with school among the students as well as variety in issues of identity that may need to be attended to.

A third challenge involves the constraints imposed by the racial and gender makeup of the teaching force in this country and the structure of the profession.

The vast majority of teachers in the United States are White, middle-class women (Howard, 1999; Nieto, 2004). This is potentially a population of teachers for whom the approaches we describe may be particularly difficult, as they likely have the most to learn about their students' communities. Although mathematics teachers are often marginalized within the broader mathematics community, they may not share the same level of marginalization with their nondominant students. Furthermore, teaching is a profession that is largely underpaid and overworked (Darling-Hammond, 1997). Most teachers, given the structure of the school day and demands on their time, have little time to conduct the kind of in-depth investigations of their students and their communities these approaches suggest. Even more important, in the broader context of increased reliance on standardized testing with high stakes for teachers and schools, these approaches that require more of teachers may be unrealistic. Any approach that argues for particular teaching strategies must take into account these very real constraints.

However, despite these challenges, we see great promise in the work of the aforementioned approaches, and we offer these critiques as a way to continue to make progress on ways to support increased equity in math classrooms. It is important to note that these approaches highlight the critical role of teachers in reproducing patterns of inequity. In the next section, we focus on the implications our review may have for the knowledge teachers need to have to best support equity and begin to "blur the lines" between cultural and domain knowledge and work simultaneously at all three levels of our model. Because of space, we do not undertake a full review of the vast literature on teacher professional development that includes a cultural or equity lens (see Sowder, 2007; Wilson & Berne, 1999). Rather, we reflect on the implications for teacher training of the research we have reviewed in this chapter, drawing on some of the relevant work in teacher professional development.

Toward this end, we briefly consider two questions. What knowledge do teachers need to know, and what professional development models might prove productive possibilities for sharing that knowledge with teachers?

## Implications for Teacher Knowledge

The work of teachers has grown considerably more complex in the past 10 years (Ball & Cohen, 1999; Cochran-Smith & Lytle, 1999; Cochran-Smith & Zeichner, 2005; Lampert & Ball, 1998; Putnam & Borko, 2000). Standards-based instructional practices require that teachers develop a specialized form of mathematics knowledge for teaching (MKT; Ball, 2005; Hill, Schilling, & Ball, 2004; Hill, Sleep, Lewis, & Ball, 2007) that reflects a particular blend of connected domain understanding with techniques and strategies to facilitate productive classroom interactions. Although the details of MKT are currently being worked out, the domain understanding required for eliciting, evaluating, and building (Carpenter, Fennema & Franke, 1996; Lampert, 1990, 2001; Schifter, 2001) on students' mathematical ideas reflects a facility with and deep understanding of mathematical concepts and procedures across the terrain of K–12 mathematics (Greeno, 1991; Ma, 1999; NCTM, 1991, 2000). Teachers need to have the opportunity to develop mathematics knowing in practice through ongoing reflection in the classroom and with their peers through the use of records of practice, video, and other learning materials (Kazemi & Franke, 2004; Lampert & Ball, 1998).

As Rochelle Gutiérrez (2002b) argues, however, knowledge of dominant mathematics must also be balanced with knowledge of how to enable students to critique the role of mathematics in society and to "contribute toward a positive relationship between mathematics, people, and society in ways that erase inequities on this planet" (p. 172). Delineating a set of teaching practices that encompasses both dominant and critical perspectives on mathematics is complicated by the fact that some classrooms are becoming increasingly diverse while others slip into hypersegregation (Orfield, Frankenberg, & Lee, 2003). Students themselves are also quite complex, as they negotiate hybrid practices, identities, and time scales through new global technologies that transcend traditional racial, social, and linguistic boundaries (Barab, Hay, Barnett, & Squire, 2001; Delpit, 2002; Gergen, 1991; Moje, Ciechanowski, Ellis, Carrillo, & Collazo, 2004). Thus, as we noted, in this chapter we do not presume to be able to comprehensively outline the knowledge teachers need to teach mathematics effectively and fairly. Instead, we juxtapose recent theoretical shifts that blur the boundaries between mathematics and cultural knowledge, with the implications of the various programs we reviewed above to propose ideas about effective mathematics teaching in classrooms with diverse populations of students.

First, the Funds of Knowledge approach would suggest that an important aspect of teacher preparation would support teachers in viewing their students as whole people with rich social and intellectual lives outside of the classroom. Activities for prospective teachers might include spending time with students and families outside of school (Civil, 2002; Foote, 2006) and bringing families into schools to better understand students' interests and skills outside of the classroom and those that exist as funds of knowledge in their communities. Additionally, professional development activities might include a study of modules developed with students' and families' funds of knowledge at the center and might offer models for alternative ways to incorporate family and community members into classroom activities. An important aspect of this work would be to support prospective or current teachers in understanding the value (both for students' learning and for social justice) of shifting the traditional power relations between families and schools and of opening communication channels.

Similarly, the activities of the Algebra Project would also suggest that supporting teachers in understanding the importance of and offering suggestions for how to better get to know the young people they are teaching is a critical focus for teacher preparation. The Algebra Project might also share an orientation for teachers that views math teaching as political activity and sees subverting current patterns of unequal access to higher mathematics as an immediate concern. Moses and Cobb (2001) argue that students need to be taught to "demand to understand" when learning mathematics, arguably placing the teacher in the role of civil rights activist. Central to this approach would be specific training in how to provide opportunities for students to physically and experientially engage mathematics, to describe mathematical relationships both in their own language and in the formal language of mathematics, and to represent mathematical ideas symbolically.

Culturally relevant pedagogy has been met with widespread appeal from educators, yet there is little research to date focusing explicitly on how to organize culturally relevant pedagogy in mathematics classrooms (although this is changing). Culturally relevant pedagogy suggests that teachers' orientation toward students is crucial—that they should hold themselves accountable for the success of *all* of their students, recognizing the capacity for success of each. In the mathematics classroom, this might mean the use of instructional strategies such as Complex Instruction (Boaler, 2006c; Cohen, 1994), where teachers attempt to disrupt traditional status hierarchies by assigning competence and fostering accountability among students. However, culturally relevant pedagogy also points to the need for a critical approach to race and privilege within mathematics teacher education (DiME, 2007; Grant & Sleeter, 2003). For White teachers to develop the cultural and intellectual awareness to engage with their African American students at a deep level, Ladson-Billings (1995) argues that they need to

- spend a significant amount of time in the African American community,
- receive structured and prolonged experiences with African American students in their preservice teaching, and
- learn how to critique our educational system in ways that inspire them to be agents of change.

This framework for teacher preparation addresses concerns raised by Sleeter (1997), who found that multiple, day-long sessions with teachers on topics in multicultural education did not provide them with sufficient depth to understand the connections between the cultural and mathematical implications of their teaching. Essential to teachers' development of culturally relevant pedagogical knowledge, then, is a shift in their perceptions about what it means to be African American in the mathematics classroom (Martin, 2000) and what is encompassed in their role as mathematics educators of African American children.

The primary aim of a social justice curriculum is to involve students directly in using mathematics to question and eventually uproot social injustice. To support this process, teachers must be trained to allow students to pose questions about local situations that they feel are unfair and to generate mathematics with their students to investigate these questions (see also *Rethinking Mathematics*; Gutstein & Peterson, 2005). This may be a chicken-and-egg situation, though, because teachers may find it difficult to build trust with students who are traditionally marginalized without first showing them that they are on their side. This also requires that teachers know how to motivate sophisticated mathematical conversations from social justice activities. And similarly, as Bartell (2005) suggests, teachers must seek out the underlying causes of unjust situations and processes to prevent reductionist discussions of cause and effect. These new roles for teachers are not easy. As Aguirre's (2007) reflections on her social justice mathematics methods course suggest, teacher resistance to change at both pedagogical and ideological levels can make it difficult for teacher educators to foster a social justice orientation to mathematics teaching.

The proliferation of equity-oriented mathematics teacher preparation and professional development programs suggests that the field is moving toward a broader understanding of how to prepare teachers for cultural diversity and justice. Enacting changes in teacher education in the ways described above, however, substantially reshapes and enlarges the boundaries of the practice of mathematics teachers. We argue, with others (Franke & Kazemi, 2001; Little, 2002), that teachers cannot do this work alone, in isolated classrooms, without the support of their peers, institutions, and mathematical and cultural *brokers* (Lave & Wenger, 1991). We also draw on Cochran-Smith and Lytle's (1999) distinction between *knowledge for practice* and *knowledge of practice* to argue that knowledge of teaching and teaching practice are not separate acts. A knowledge-of-practice perspective assumes that teachers' knowing emerges from their participation in teacher and other (cultural, ethnic, racial, socioeconomic) communities and is connected to their practice in relation to broader sociopolitical processes and institutions.

## CONCLUSION

We reviewed literature relevant to what we consider to be a critical area for mathematics education research—the cultural nature of mathematics teaching and learning and the ways in which we maintain or blur boundaries between cultural knowledge and domain knowledge in mathematics. In doing so, we have explored both research that focuses on relations between cultural and domain knowledge and research that examines issues of race and culture inside of math classrooms. It is interesting to note in our review that often, contributions to our understanding of mathematics teaching and learning have not explicitly attended to issues of race and culture, and contributions to our understanding of the relation between domain and cultural knowledge often do not stem from the study of mathematics classrooms.

The reasons often given for this disjuncture—that our society views mathematics as culture free or that mathematics education researchers are not concerned with issues of race and culture in the learning of mathematics—do not capture the complexity of the situation. Although these explanations may very well be valid, we would also like to offer the following possibilities:

1. Mathematics may be a particularly challenging domain to map students' everyday cultural practices onto, as its very purpose is to abstract and generalize (rather than to reflect on the details of any particular experience). This is different from an argument that math is culture free; rather, culture is less visible from a mathematical lens.
2. Mathematics education researchers have yet to develop and agree on methods that can be used to document cultural practices and processes within mathematics classrooms and systems. That is, we simply have few conceptual and practical

tools to understand issues of culture in mathematics classrooms and even fewer models of how to account for multiple levels of culture, race, and access simultaneously (Nasir & Hand, 2006).

3. Mathematics holds a privileged status in our society as an elite activity for the smartest of citizens. That assumption supports a view of math as out of the reach of the "common" man and thus disconnected from and inaccessible through everyday experiences.

Given these ideological and practical constraints, mathematics education research in general remains underdeveloped with respect to issues of cultural versus mathematical knowledge. However, study of the intersections of cultural and domain knowledge in mathematics may push the field of educational research in productive directions. In our view, this would mean attending to some of the issues that we have outlined in this chapter, including making connections between everyday and mathematical ideas, revisiting what counts as mathematics, who makes the decisions about what counts (and about whose knowledge is privileged), and how these play out in the classroom in terms of the practices, roles, materials, and tools authorized for mathematical activity.

We would like to highlight what we view as critical tensions and issues that could serve as directions for research and practice.

First, many of the scholars whose work we review in this chapter, social justice perspectives in particular, argue that given that all mathematical problems reside in some sort of context—particularly at the lower grades—we need to make the context reflect the realities of students' lives. Furthermore, we cannot impose these contexts on the basis of racial group membership or social class categories; rather, we must build them with students through conversations and shared experiences. This perspective raises a set of questions. Is it important to make the context relevant (e.g., creating social justice tasks), or the structures for classroom participation (e.g., fostering multidimensional classrooms)? How are they different? Is one better for a particular group of students? How is developing social justice within the mathematics classroom (or what Boaler, 2006b, calls "relational equity") related to the development of critical mathematics literacy? Embedded in these concerns are varying assumptions about the purpose of mathematics education—assumptions that we must lay bare and investigate.

Second, another major thrust in the work that we reviewed is intersubjectivity as an important aspect of math learning and "third space" (K. D. Gutierrez et al., 1995) as a potential way to conceptualize the blurring of cultural and domain knowledge in mathematics classrooms. How does a third space (which is related to hybrid discourse structures) develop in mathematics classrooms, and how does it relate to the type of engagement in mathematics available to nondominant students? As classroom participants work toward understanding the meanings of the mathematical context, content, method, or representation produced by others, how are these meanings negotiated? What does it mean to foster dialogical inquiry within a culturally diverse classroom? Will questions about who is making these meanings and what is the social and cultural context of their meaning-making system begin to emerge?

Third, given all of the research that we have reviewed, what can we glean with respect to how to best prepare students (and especially urban students) to productively engage in mathematics classrooms and high-level mathematics? One aspect of this involves thinking about how to incorporate students' voices and experiences into the math classroom. We want to draw a distinction between using everyday cultural knowledge (for instance, the basketball players' knowledge about average and percentage) as a point of entry into a mathematical discussion and using it to limit what students can learn to what they already know. We strongly support the former usage and argue that the math knowledge students accrue in everyday practices should be used as both a conceptual and a social lever to support students' deeper engagement in math and their identities as capable math learners, not as a limit on their ability to engage abstract mathematics.

Finally, our review raises important concerns about what mathematics to cover in schools. What are the constraints and affordances of a "mathematics-for-all" approach? How do we think about this in the context of the current NCLB climate of standards, high-stakes testing, and threats of state takeovers? What are the compromises involved in creating specialized mathematics instruction for groups of students? Would such an approach lead to artificial distinctions and stereotypes?

In our view, the tripartite model that we offered at the beginning of the chapter may offer some traction in thinking about and making progress on these critical issues. Keeping in view the three levels at which math learning and culture intersect may offer us a way to conceptualize the multiple, simultaneous ways that math learning and culture become intertwined in math classrooms. More specifically, the model points to the intertwined nature of concerns with math knowing as a cultural activity, math learning as a cultural enterprise, and math education as a cultural and political activity. With respect to math knowing as a cultural activity, we have highlighted the importance of building on what students know and on understanding how they express what they know. This is deeply connected to viewing math learning as a cultural enterprise and broadening what it means to learn math and be a mathematics learner. This broadening is related to issues of identity—recognizing that students have to negotiate membership across different social contexts (including math) and creating opportunities for them to make the practice their own. Finally, we have argued for the critical nature of understanding mathematics as a cultural system and using mathematics as a tool of empowerment and awareness in issues of social justice (both in the classroom and more broadly through more careful analysis of the outcomes of NCLB).

We would like to return briefly to reflect on the example of the solution strategies of the basketball players with which we opened the chapter. We made sense of this example in the introduction by arguing that it illustrates the distinction the players made between the math of everyday life and the math of school. In some ways, it could be argued that this example, then, reflects the first level of our model—math knowing as cultural activity—as it is concerned with the way the individual student is developing knowledge of mathematics across practices and not challenging the disjuncture between the two.

But the pattern of solutions we encountered also reflects students' experiences of mathematics and mathematics classrooms as places where they are not central participants, where they are not constructors of knowledge, and where "smart" responses mean applying algorithms. These experiences left many of them with the view that Boaler and Greeno's (2000) calculus students articulate: that math does not have to make sense and does not have to be personally meaningful. This corresponds to the second level of the model, that learning in math classrooms is a cultural enterprise with particular norms, values, and appropriate stances and activities.

This example also illustrates the third aspect of the model, that math education itself is a cultural (and indeed political) activity. The basketball players' responses reflect their social and political position in our society—as urban African American young men. They are not afforded access to mathematics teaching or resources at the school level that might allow them to use mathematics to challenge existing hierarchies and injustices and at the same time to create more prosperous futures for themselves and their families.

This last point highlights the ways in which our task, with respect to the preparation of students from nondominant groups and students from impoverished communities, is multipronged. We must prepare urban students to simultaneously challenge existing hierarchies of knowledge and to be competitive in a system that relies on such hierarchies. This involves both relatively long-range and short-term goals. Again, this cannot happen simply through lone teachers acting in isolated classrooms—rather, it must be a part of a collective effort on the part of researchers, educators, and policymakers. It must involve a paradigm shift with respect to the purposes for teaching mathematics and the desired outcomes. When these shifts occur, multiple cultures will be part and parcel of the math classroom, and no longer will the boundary between domain and cultural knowledge be constructed so forcefully by our collective assumptions.

## NOTES

[1]Gloria Ladson-Billings (2006) has cast this as a national educational debt to highlight that these are socially and historically located trends, not merely differences in scores between individuals.

[2]We consider culture and approaches to teaching math in the next section.

[3]Similar arguments were successful in passage of Head Start funding.

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